ODEs - Math 365
Exam 2

Name:____

1. Consider the initial value problem: $6y'' - 5y' + y = 0, \qquad y(0) = 4, \qquad y'(0) = 0.$

   (a) Solve the problem.

   (b) Describe the behavior as $t \to \infty$.

2. Let $f(t) = 2 - e^{3t}$ and $g(t) = e^{6t} - 4$. Determine whether f and g are linearly independent or linearly dependent.

3. Consider the differential equation: $y'' + 2y' + 2y = g(t)$.

(a) Find the general solution to the homogeneous equation.

(b) Let $g(t) = t + \cos(2t)$. Find a particular solution.

(c) With $g(t)$ defined as in part (b), what is the general solution to the differential equation.

4. A mass weighing 4 lb streches a spring 1 inch. The mass is attached to a viscous damper with a damping constant of 2 lb-sec/ft. If the mass is set in motion from its equilibrium position with an upward velocity of 2 ft/sec, find its position u at any time t.

5. Find the general solution to the differential equation: $9y'' - 12y' + 4y = 0$, and describe its behavior.

6. Use the method of reduction of order to find a second solution to the differential equation. **Note:** You may leave the second solution in integral form.

$$y'' - ty + y = 0, \qquad y_1(t) = t$$

7. Consider the differential equation $L[y] = ay'' + by' + cy = 0$.

   (a) Find the characteristic equation.

   (b) Assume that the determinant of the characteristic equation is positive. Describe the three distinct types of solutions that can occur in this situation.
   Note: It is not necessary to assume anything about the constants c_1 or c_2.
   Note 2: You may "buy" a hint on this problem for 3 pts.

8. Formally prove the following: For constants a, b, and c, if $y_1(t)$ and $y_2(t)$ are solutions to $L[y] = ay'' + by' + cy = 0$ then $c_1y_1(t) + c_2y_2(t)$ is a solution to $L[y] = 0$.