

Name: _____

1. Classify both of the following ODEs. For only one of them, sketch the direction field for $t > 0$ and describe the end behavior. If you choose the second one, you lose two points.

1. $\frac{dy}{dt} + y^2 = t$

2. $\frac{dy}{dt} = 5 - 3y$

2. Solve the following ODE. Your solution need not be explicit. You may buy a one word hint for this problem for three points.

$$\frac{dy}{dx} = \frac{y \cos x}{1 + 2y^2}, \quad y(0) = 1$$

3. Find the general solution to the following ODE. A two step hint is available for three points.

$$ty' + (t + 1)y = t, \quad t > 0$$

4. Consider the following ODE: $ay'' + by' + cy = 0$, where a , b , and c are constants.

(a) What is the characteristic equation?

(b) Let r_1 and r_2 be roots of the characteristic equation. Give the three possible cases for the resulting general solution. I will tell you the three cases for three points.

5. (a) Verify that $y_1 = e^t$ and $y_2 = te^t$ are solutions to $y'' - 2y' + y = 0$.
- (b) Prove either that y_1 and y_2 form a fundamental set of solutions or that they do not form a fundamental set of solutions.
6. Consider $y'' + 2y' + y = 3e^{-t}$. A fundamental set of solutions to the homogeneous equation is $y_1 = e^{-t}$ and $y_2 = te^{-t}$. Set up the integral that you would use to find a particular solution if you were using variation of parameters. Do not solve. You can buy the formula for four points.

7. (a) Find the radius of convergence for the power series $\sum_{k=0}^{\infty} \frac{k}{3^{k+1}} x^k$.

(b) What does it mean to say that a function is **analytic** at a point $x = x_0$

8. (a) Find the recurrence relation for the series solution of the differential equation about the given point:

$$(2 + x^2)y'' - 4y = 0, \quad x_0 = 0$$

(b) Let $a_0 = 0$ and $a_1 = 1$. Find the first three nonzero terms of this solution.

9. Determine a lower bound for the radius of convergence of series solutions about $x=2$ for the equation

$$(x + 1)(x^2 + 4)y'' - 2\pi xy' - e^2 y = 0.$$

10. Find the solution to the initial value problem

$$4x^2 y'' + y = 0, \quad y(1) = 3, \quad y'(1) = 2$$

Describe how the solution behaves as $x \rightarrow 0$.

11. Discuss the nature of the solutions of the equation

$$2x(2+x)y'' + y' - xy = 0$$

near the singular point $x = -2$. Specifically, show that $x = -2$ is a regular singular point. Find the indicial equation and the exponents at the singularity. Write down the form of two independent solutions and describe their behavior as $x \rightarrow -2$.