

Math 365.1

Quizzes

Name: _____

Quiz 1

1. Sketch a direction field for the given differential equation. Determine the behavior of y as $t \rightarrow \infty$. If this behavior depends on the initial value y_0 describe the dependency.

$$y' = 1 + 3y$$

2. Write a differential equation of the form $\frac{dy}{dt} = ay + b$ such that all other solutions diverge from $y = 1$.

Quiz 2

3. Classify the following differential equation (ode/pde, order, linear/nonlinear)

$$t^2 y'' + 5ty' + (\sin t)y = 0$$

4. Verify that $u(x, t) = \sin(x - at)$, λ a real constant, is a solution of the given partial differential equation:

$$a^2 u_{xx} = u_{tt}$$

5. Find the general solution to $y' - 2y = t^2 e^{2t}$

Quiz 3

6. Find the solution to the following initial value problem in explicit form:

$$\frac{dy}{dx} = \frac{2x}{y + x^2 y}, \quad y(0) = -2$$

7. A trust fund is established for the daughter of a wealthy man with an initial amount of \$100,000. The fund collects interest at an annual rate of $r = 0.08$. The funds are dispersed to the daughter at a rate of \$1000/month. Find a differential equation that represents the change in the value of the fund. Assume continuous compounding and continuous withdraws. How long will it take for the trust fund to run out of money?

Quiz 4

8. Determine (without solving the problem) the largest interval on which the solution to the following initial value problem is certain to exist.

$$(\ln t)y' + y = 0, \quad y(1/2) = 7$$

9. For the following equation, sketch $f(y)$ versus y , determine the critical points and classify them as asymptotically stable, unstable, or semistable. In addition, sketch the solutions for $y(0) = 0$, $y(0) = 3.5$, and $y(0) = 5$.

$$\frac{dy}{dt} = t^2 - 6t - 8$$

Quiz 5

10. Find the solution to the initial value problem and sketch its solution. In particular, show what happens as $t \rightarrow \infty$.

$$y'' + 8y' + 15y = 0, \quad y(0) = 1, \quad y'(0) = 5$$

11. Find the general solution to the ODE. How will the solution behave as $t \rightarrow \infty$. Does this behavior depend on the initial conditions?

$$-2y'' - 5y' + 12y = 0$$

Quiz 6

12. If the Wronskian W of f and g is t^2e^t and if $f(t) = t$, find $g(t)$. Though not necessary, assume $t > 0$.
13. Show that the functions $f(t) = t^2$ and $g(t) = t^3$ are linearly independent.
14. Show that the functions $f(t) = t - 3$ and $g(t) = 9 - 3t$ are linearly dependent.

Quiz 7

15. Find the solution to the given initial value problem. Describe its behavior for increasing t :

$$y'' + y' + \frac{5}{4}y = 0, \quad y(0) = 3, \quad y'(0) = 1$$

16. (a) Find the general solution to the differential equation $9y'' - 6y' + 1 = 0$.
- (b) Let $y(0) = 1$, $y'(0) = b$. Find the critical value b that separates solutions that tend to infinity from solutions that tend to negative infinity.

Quiz 8

Consider the ODE $2y'' + y' - 6y = g(t)$

17. Use the method of undetermined coefficients to find the general solution to the above ODE if $g(t) = t^2 - 1$.
18. If $g(t) = t^2e^{3t} + \sin t$, explain very briefly how you would use the method of undetermined coefficients to solve the ODE.
19. Let $g(t) = 3e^{2t}$. Set up the variation of parameters method of solution.

Quiz 9

20. Determine the radius of convergence of $\sum_{n=1}^{\infty} \frac{(x+1)^n}{n2^n}$.

21. Rewrite the given expression as a sum whose generic term involves x^n .

$$\sum_{n=1}^{\infty} na_n x^{n-1} + x \sum_{n=0}^{\infty} a_n x^n$$

Quiz 10

22. Find the recurrence relation for the series solution of the differential equation about the given point:

$$y'' - xy' - y = 0, \quad x_0 = 0$$

23. Assume $a_0 = 1$ and $a_1 = 0$. Find the first four terms of this solution.

Quiz 11

24. Consider the differential equation: $2x^2y'' + 3xy' + (2x^2 - 1)y = 0$

- (a) Show that $x = 0$ is a regular singular point.
- (b) Find the indicial equation at $x = 0$ and use it to determine the exponents at the singularity.
- (c) Find the recurrence relation.
- (d) Find the first three terms of two linearly independent series solutions about $x = 0$.
- (e) Describe the solution behavior as $x \rightarrow 0$.

25. Without solving the given ODE, determine all regular singular points, the indicial equation, the exponents at the singularity, and the qualitative behavior as $x \rightarrow 0$.

- (a) $x^2y'' - x(2 + x)y' + (2 + x^2)y = 0$
- (b) $(x + 1)^2y'' + 3(x^2 - 1)y' + 3y = 0$