ODEs - Math 365
Exam 1
February 15, 2005

Name:______________________________

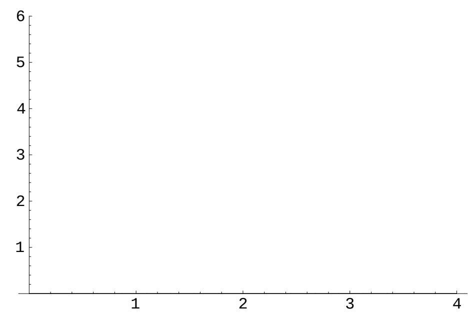
1. Classify the following differential equations using the following descriptors: 1) linear vs. nonlinear, 2) order, and 3) partial vs. ordinary.

   (a) $5x^2y'' - 3e^y = 2x^2 - 7$

   (b) $-\epsilon(u_{xx} + u_{yy}) + b_1 u_x + b_2 u_y + gu = 0$

   (c) $(3t^2 - \sin t)\dfrac{d^4y}{dt^4} - (e^t - 7)\dfrac{d^2y}{dt^2} - (\ln t)y = 0$

2. Draw a direction field for $\dfrac{dy}{dt} = (t-2)(y-t), \quad t, y \geq 0$. Are there equilibrium solutions? Explain approximately what the end behavior is and how the initial value $y(0) = y_0$ comes into play. Note: **Do not** attempt to find a solution or some specific value of y_0 to answer this question. Just give a broad idea of what will happen.



3. Verify that $\phi(x) = (x-2)^{3/2} + x^2$ is a solution to $4(x-2)^2y'' - 3y = 5x^2 - 32x - 32, \quad y(3) = 10,\ y'(3) = 15/2$. Determine the interval for which the solution is valid.

4. Consider a model for the population of trout in thousands. In class, we considered the logistic growth model given by

$$\frac{dy}{dt} = r(1 - \frac{y}{K})y, \qquad r, K > 0,\ y \geq 0 \tag{1}$$

Equation (1) can be modified to model the fish population at a hatchery where fish are harvested at a rate h, via

$$\frac{dy}{dt} = r(1 - \frac{y}{K})y - h \qquad r, K, h > 0,\ y \geq 0 \tag{2}$$

(a) Let $r = 4$ and $K = 4$. Sketch $f(y)$ versus y for model (1). Include concavity in your sketch. Find and classify the equilibrium points.

(b) In addition, let $h = 3$. Include concavity in your sketch. Sketch $f(y)$ versus y for model (2). Find and classify the equilibrium points.

(c) Sketch possible solutions for both models including concavity for each possible region of y_0.

(d) Explain in simple (but proper) English what each model means in regards to the population of fish and how they differ.

5. (a) Consider $y' = 5te^y + 3, \quad y(3) = -2$. Use a change of variables to transform this equation into an equation in z and τ with initial condition $z(0) = 0$.

(b) Given $y' = 3t^2 + y, \quad y(0) = 0$. Let $\phi_0(t) = 0$ and use Picards iteration technique to find $\phi_2(t)$.

6. Consider $y' = \dfrac{(t+2)y}{y+3}$.

(a) Determine all values (t_0, y_0) for which the hypotheses for Theorem 2.4.2 hold.

(b) Solve the problem given $y(0) = 1$. **Do not** attempt to solve explicitly for y.

(c) Is there a contradiction between your answers for (a) and (b). Explain.

7. Consider $(x-2)y' + 2y = \ln x$.

(a) Determine all intervals on which a unique solution is guaranteed to exist by applying the appropriate theorem.

(b) Solve the problem given $y(1) = 2$.

8. Find the general solution for $3x^2y^2 + 2x^3yy' - (\sin y)y' = 0$.