ODEs - Math 365
Exam 2
April 5, 2005

Name:____

1. (5) Find the general solution to $y'' - 2y' + 10y = 0$

2. (10) Solve $4y'' - 12y' + 9y = 0, \quad y(0) = 2,\ y'(0) = -1$ and determine the solution's end behavior.

3. (15) Consider $y'' - 5y' + 6y = e^{-t}$

(a) Find the solution to the homogeneous equation.

(b) Show your solutions are linearly independent.

(c) Use variation of parameters to find the general solution.

4. (5) **FORMAL** proof: For this question you will be graded on the form, style, and succinctness of your proof at least as much as on the correctness of the proof.
Let $ay'' + by' + cy = 0$. Prove that if $y = e^{rt}$ is a solution, then r is a root of the characteristic equation.

5. (10) Consider the ODE $y'' - ty' + y = 0$. Given $y_1 = t$ is a solution, use the method of reduction of order to find a second solution (leave your second solution in integral form).

6. (15) In class we only investigated unforced vibration. However, you have all the tools to consider a forced vibration problem as well. To wit, a mass weighing $\frac{8}{3}$ lbs. displaces a spring $\frac{32}{3}$ feet. The mass is attached to a viscous damper with a damping constant of $\frac{1}{3}$ lb-sec/ft and is acted upon by a periodic external force of $\frac{5}{6}\cos t$. If the weight is pulled down an additional foot and then pushed with a velocity of 4 ft/sec, find the position function for any time t.

(a) Set up the differential equation.

(b) Find the solution to the homogeneous equation.

(c) Find a particular solution to the nonhomogeneous equation.

(d) Write the equation with only one cosine and describe it's behavior. In particular, comment on the effects of the unforced (homogeneous) part of the solution (called the transient solution) and the forced (nonhomogeneous) part (called the steady-state solution).