Math 365, Final Exam, 5/3/05

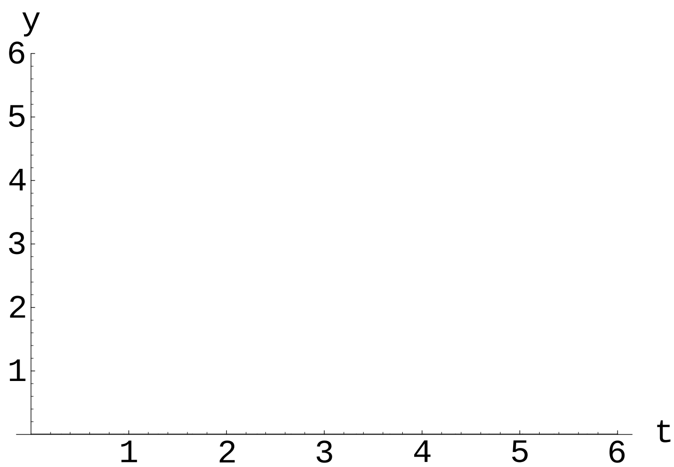
Name:____

1. (a) Classify the following differential equations using the following descriptors: 1) linear vs. nonlinear, 2) order, and 3) partial vs. ordinary.

   i. $u_{tt} - u_{xxxx}$ (Beam equation)

   ii. $y''' = 2\sin x + (y')^2$

   (b) Draw a direction field for $\frac{dy}{dt} = (y-2)(y+t-4), \quad t, y \geq 0$. Sketch plausible solutions for initial conditions $y_0 = 1, y_0 = 3$, and $y_0 = 5$.



2. Determine a lower bound for the radius of convergence of series solutions about $x = 5$ for the equation

$$(x+1)(x^2+9)y'' - (x^2-2x-15)y' + 2x^2y = 0$$

3. Find the solution of the following initial value problem. (Hint=3pts)

$$ty' + 2y = t^{-1}\cos t, \qquad y(\pi/2) = 0,\ t > 0$$

4. Find an explicit solution to the given initial value problem. (Hint=3pts)

$$9x^2 + y - 1 + (x - 4y)y' = 0, \qquad y(1) = 0$$

5. (15 pts)Consider the following ODE with solutions $y_1(x) = x$ and $y_2(x) = \sin x$

$$(1 - x \cot x)y'' - xy' + y = 0, \quad 0 < x < \pi$$

(a) Verify that $y_2(x)$ is a solution.

(b) Show that y_1 and y_2 form a fundamental set of solutions.

(c) What is "fundamental" about a fundamental set of solutions, i.e. what's the big deal?

6. (a) Find the radius of convergence for the power series $\sum_{k=0}^{\infty} \frac{3k}{2^{k+2}} x^k$.

(b) What does it mean to say that a function is **analytic** at a point $x = x_0$?

7. Find the general solution to each of the following ODE's

(a) $y'' + 2y' + 6y = 0$

(b) $9y'' + 6y' + y = 0$

8. Determine the general solution of the following differential equation that is valid in any interval not containing the singular point.

$$x^2y'' + 3xy' + 5y = 0$$

9. (a) Find the recurrence relation for the series solution of the differential equation about the given point.

$$(4 - x^2)y'' + 2y = 0, \qquad x_0 = 0$$

(b) Let $a_0 = 1$ and $a_1 = 0$. Find the solution (ie. find all terms of the power series - Note: there are only a few nonzero terms).

10. Consider the ODE, $y'' + 2y' + ky = g(t)$.

(a) For $g(t) = 0$, what values of the constant k will guarantee that $y \to 0$ as $t \to \infty$?

(b) Find the general solution when $k = -3$ and $g(t) = 2e^{3t}$.

11. Determine the smallest positive singular point of the Chebyshev equation (below) and determine whether it is regular.

$$(1 - x^2)y'' - xy' + \alpha^2 y = 0$$