

Calc III - Math 311
Exam 1
June 18, 2008

Name: _____

Show work if you desire partial credit. Circle or box your final answers where appropriate. Questions worth 10 points except where noted.

1. (5pts) Find a unit vector in the direction of $\langle 3, -6, 4 \rangle$.

2. (15pts) Let $\mathbf{a} = \langle 3, -1, 5 \rangle$ and $\mathbf{b} = \langle 2, -2, 5 \rangle$. Find

(a) $\mathbf{a} \cdot \mathbf{b}$

(b) $\mathbf{a} \times \mathbf{b}$

(c) $\text{proj}_{\mathbf{a}} \mathbf{b}$

3. (a) Give parametric equations for the line that passes through $(1, 3, 5)$ and $(-3, -7, 2)$.

(b) (5pts) Give the equation for the plane that contains this line and the origin.

4. (5pts) Find all intervals of continuity of $\mathbf{r}(t) = \langle \frac{3}{t}, \sqrt{5-t} \rangle$.

5. (5pts) Find the unit tangent vector of $\mathbf{r}(t) = \langle 3 \sin 2t, e^{3t} \rangle$.

6. Find the curvature of $\mathbf{r}(t) = \langle -\cos t, t, \sin t \rangle$.

7. (18pts) A bowling ball of mass 1 kg. is shot from a cannon towards the east at an angle of $\theta = \pi/4$ with an initial speed of $v_o = 200$ meters/sec and initial height 20 meters. The spinning of the ball creates a southerly Magnus force of 1 Newton.

(a) Find the landing location (stated in meters east and meters south).

(b) Find an expression representing the total distance traveled through the air by the bowling ball.

8. (7pts) Find the distance between the point $(3, -3, 2)$ and the plane $2x - 9y - z = 2$.

9. (5pts) Explain why the volume of a parallelepiped equals $|\mathbf{c} \cdot (\mathbf{a} \times \mathbf{b})|$ using the fact that

$$|\mathbf{c} \cdot (\mathbf{a} \times \mathbf{b})| = \|\mathbf{a} \times \mathbf{b}\| \frac{\mathbf{c} \cdot (\mathbf{a} \times \mathbf{b})}{\|\mathbf{a} \times \mathbf{b}\|}.$$

Draw a picture!