Calc III - Math 311
Exam 2
June 25, 2008

Name:_____

Show work if you desire partial credit. Circle or box your final answers where appropriate. Questions worth 10 points except where noted.

1. Show the first limit does not exist. Show the second limit exists and find its value.

   (a) $\lim_{(x,y)\to(0,0)} \dfrac{5x^{1/2}y^2}{x-3y^4}$

   (b) $\lim_{(x,y,z)\to(0,0,0)} \dfrac{x^2y^2z^3}{x^4+5x^2yz+z^4}$

2. (5pts) Find the domain of the function $f(x, y, z) = \dfrac{3\sqrt{x-2}}{(1-e^y)\ln z}$.

3. (5pts) For $f(w, x, y, z) = 3w^2z - x^3e^y - \cos(3w + y)$, find $\nabla f(w, x, y, z)$.

4. Let $f(x, y) = x^3y - \sqrt{y^3} + 2x$. Find the linear approximation function for f at the point $(2, 4)$ and use it to approximate $f(2.1, 3.8)$.

5. (5pts) Consider the relationship defined by $x^2 + 3xyz + y^2z = 0$. Find $\dfrac{\partial z}{\partial y}$.

6. In section 13.7, we will investigate spherical coordinates. The conversion equations are $x = \rho \sin\phi \cos\theta$, $y = \rho \sin\phi \sin\theta$, and $z = \rho \cos\phi$. Consider the function $f(x, y, z)$ with x, y, and z defined as above.

   (a) Find an expresion for f_ϕ.

   (b) Define $H = f_\phi$ from part (a). Find an expression for $f_{\phi\phi}$ using H and Liebnitz notation. In other words, don't actually take any derivatives for this part.

7. Let $f(x) = 2xy + x^2y^2$. Find $D_{\mathbf{u}} f(-2, 3)$ in the direction from the point $P(1, 2, 1)$ to $Q(3, 1, 0)$. What is the maximum rate of change of f and in which direction does it occur?

8. Find and classify all critical points of the function $f(x, y) = 3xy - x^3 - y^3$.

9. Maximize the function $f(x, y) = x^3y^2$ subject to the constraint $x^2 + y^2 = 4$.

10. (5pts) The method of steepest ascent is a numerical method for finding a local maximum of a function. You begin with an initial guess. You then travel in the steepest direction until you come to a point at which you are no longer increasing. You then take this point and repeat the process. With the cotour plot below, trace out the method of steepest ascent for two or three iterations, beginning at the initial point $(0.15, 2)$ indicated.