Calc III - Math 311 - Final Exam - July 11, 2008

Name:____

Show work if you desire partial credit. Circle or box your final answers where appropriate. Questions worth 10 points except where noted.

1. Let $\mathbf{a} = \langle 2, 3, 5 \rangle$ and $\mathbf{b} = \langle -1, 5, 7 \rangle$.
   (a) If $\mathbf{a} \cdot \mathbf{b} = 0$ what do you know?
   (b) If $\mathbf{a} \times \mathbf{b} = 0$ what do you know?
   (c) Find $\mathbf{a} \cdot \mathbf{b}$.
   (d) Find $\mathbf{a} \times \mathbf{b}$.

2. Let $\mathbf{F} = \langle y, x^2 y, 3z + y \rangle$
   (a) If div $\mathbf{F} = 0$ what do you know?
   (b) If curl $\mathbf{F} = 0$ what do you know?
   (c) Find div $\mathbf{F}$.
   (d) Find curl $\mathbf{F}$.

3. Consider $f(x, y) = \dfrac{\sqrt{xy^2}}{x^2 - y}$.

   (a) Find the domain of f.

   (b) Find $f_x(x, y)$ (Do not simplify).

4. Show $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy^2}{x^2 + y^4}$ does not exist.

5. Let $f(x, y) = x^2 \sin 4y$. Find $D_{\mathbf{u}}f(x, y)$ where $\mathbf{u}$ points in the direction from $(-2, \pi/8)$ to $(0, 0)$.

6. Locate and classify the critical points of $f(x, y) = e^{-x^2}(y^2 + 1)$.

7. Evaluate $\displaystyle\int_0^{\pi} \int_0^{2} y \sin(xy)\, dx\, dy$.

8. Evaluate $\displaystyle\int\int_R \sqrt{x^2 + y^2 + 1}\, dA$ where R is the disk $x^2 + y^2 \leq 16$.

9. Evaluate $\iiint_Q (x^2 + y^2 + z)\, dV$ using cylindrical coordinates where Q is the region between $z = -\sqrt{x^2+y^2}$ and $z = \sqrt{x^2+y^2}$ and inside $x^2 + y^2 = 4$.

10. Evaluate the line integral $\int_C (3x - y) ds$ where C is the quarter circle on $x^2 + y^2 = 9$ from $(0, 3)$ to $(3, 0)$.

11. Show that the line integral $\int_C (2xe^{x^2} - 2y)dx + (2y - 2x)dy$ where C runs from $(1, 2)$ to $(-1, 1)$ is independent of path and use a potential function to evaluate it.

12. Use Green's theorem to evaluate $\int_C (xy - e^{2x})dx + (2x^2 - 4y^2)dy$ where C is formed by $y = x^2$ and $y = 8 - x^2$ oriented clockwise.

13. Evaluate the surface integral $\int\int_S (z - y^2) dS$ where S is the portion of the parabaloid $z = x^2 + y^2$ below $z = 4$.

14. Use Stoke's Theorem to evaluate $\int\int_S (\nabla \times \mathbf{F}) \cdot \mathbf{n}\, dS$ where $\mathbf{F}(x, y, z) = \left\langle e^{z^2}, 4z - y, 8x \sin y \right\rangle$ and where S is the portion of the paraboloid $z = 4 - x^2 - y^2$ above the xy-plane, oriented so the unit normal vecotrs point to the outside of the parabaloid.